Grids on the Earth for Sampling, Modeling, and Visualization:

Opinions, Personal Experience, and Pointers

By

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1. Equal Areas

Area is an important consideration in studies focusing on land cover, species living space, and numerous other topics related the earth surface. When area is an important consideration it makes sense to use map projections yielding equal area maps.

When it comes to developing and using equal areas grids there has been substantial resistance. A case in point is NASA's continued used of equal angle grids for its level three satellite products. The reasons for resistance to equal areas grids are numerous. The human conceptual model of latitude and longitude is firmly entrenched, both in terms of historical precedent and in terms of contemporary usage and instruction. Further the data processing infrastructure for high throughput production of equal angles maps is proven and stable. NASA can also argue that really rigorous scientific analysis uses level 2 data. (Of course part of this relates to equal angle grids being unsuitable for polar or global modeling. Resampling from equal angle grids to produce grids suitable grids for polar or global modeling involves substantial information loss.)

The observation that area is important is general accepted. NASA would likely consider implementing an equal area grid methodology if it had some desirable properties and widespread community support. Keys features of equal angle cells seem to be related to aggregation and nice boundaries. A "square" 5 x 5 grid of one degree cells can be combined into 5 degree cell and the cell boundaries on the earth are "nice." Small neighboring equal areas regions on the earth can be aggregated in various ways. It is easy to do from equal area map projects, but boundaries are not like to be nice in terms for great circle segments. At what point does equal area become more important the nice boundaries?

A next step we might even consider increases in area related to mountains, valleys and multilevel buildings. A further step might include further surface irregularities at smaller scales that are relevant to the phenomena in interest. A fractal assessment of area increases as we look more and more closely. However after some scale the additional area is irrelevant in our conceptual models. For a 3-D analogy, ducks don't fit well in nanotubes

2. Using Round Regions

In science it would often seem advantageous to use round regions. Consider data description for earth surface using regions of 1 square meter. Suppose we were to choose 1 square meter regions to be monitored from an equal area map projection. We could chose rectangles with height .000001 meters and width 1000000 meters. Why not?

I don't think most people would even consider the option. Partly this is because alternative of a meter square is cognitively simpler and more familiar. Secondarily, ignoring the measurement difficulties associated with the region height, scientists would be concerned about the spatial variability over 1000000 miles that would to need understood. A multispectral sweep could be composed of reflectance from tiny parts of animal, mineral, and vegetable entities. The lack of homogeneity would be problematic for scientific modeling and understanding.

Waldo Tobler's (1970) first law of geography is that "Everything is related to everything else, but near things are more related than distant things." We may be aware of a few exceptions distant patterns are important locally. Changing in the Southern Oscillation Index can signal changes that are seem far in space and time. However, for the most part Tobler's law reflects our experience

A criterion for assessing the roundness of convex polygons in 2-D or polytopes in 3-D is the second non-dimensional moment about the center. This is described in Conway and Sloane 1998 *Sphere Packing Lattices and Groups.* Think of the second moment as the average distance of each point in the region on map projection from the region center. Since distance from the center involves squaring the x and y differences, extreme values for either difference lead to a large distance. For a fixed area in 2-D, a circle avoids large distances. Non-overlapping circles are not area filling. Hexagons provide the "roundest" regular 2-D area filling regions.

3. Global Grids and Sampling on the Earth

Tony Olsen, of USEPA, sought to develop coherent sampling schemes for the surface of the earth. He sought to use equal area grids and was particularly interested in hexagon grids. He had seen them from me in bivariate density plots. I had used hexagon grids to summarizing his data from acid rain monitoring sites and to summarize numerous reported studies of the radioisotopes in milk after Chernobyl. Researcher before and after me have been drawn to the desirable features of equal area hexagons. These include at least one key environmental researcher in Tony's research community.

Tony enlisted John Kimerling and a student, Kevin Sahr, to develop of a hexagon grid systems for the surface of the earth. Tony also sought the input/guidance of other cartographers such as Waldo Tobler and Michael Goodchild.

In one meeting Michael Goodchild got the attendees to list desirable properties for a grid. This would provide a basis for comparing different proposed Global grids. The list of desirable properties included equal area and implicitly round regions. This item favored hexagons. Well thought out grids were proposed. I liked the Kimerling-Sahr approach. described below.

The Kimerling-Sahr approach started with an equal area projection of the earth onto an icosahedron. They enlisted the well-known USGS cartographer John Snyder to come out of retirement briefly to address slivers in currently available projection. The icosahedron (composed of 20 triangles) could be cut at triangle edges and laid flat. Kevin could embed a hexagon in each of the 20 triangular faces and the three left over small triangles at the vertices of the 20 triangles would fit together the make 12 pentagons. Rotating and reducing the hexagon at the center of a triangle and filling in adjacent hexagons of the same size led to a grid with 80 hexagons and 12 pentagons and so on. (Some helpful figures are in a ISEA Global Grids newsletter article available in the class links). The icosahedron could be oriented so the 12 singularities were in the ocean and the icosahedron could be made symmetric about the equator.

Having seen one of Kevin's visualization examples, I decide to develop methodology in Splus and produce a higher resolution example with 196,820 hexagon + 12 pentagon.) Kevin's sent me his binning software to adapt and the Etopo 5' elevation/depth data. I produced a picture of the world that would fold into an icosahedron with gluing tabs. John Kimerling provided the elevation color ramp. This was selected for a GMU Art and Science exhibit. It was omitted from the pdf of the ISEA newsletter article due to its size. A link will be provided on the new class web were there is more space.

John won an Oregon State award for his work. The award allowed him to invite speakers to a conference/celebration at Oregon State. Ralph Kahn's (JPL) talk addressed what grids could provide, their merits and problems. I learned a lot from him. For example there is a serious loss of information in resampling from equal angle grids to obtain grids useful for global or polar modeling. Someplace (perhaps not in his talk) I learned that the approximates to Navier-Stokes equations work out better on hexagon grids than square grids.

For my talk I developed the bookkeeping for neighbor to neighbor smoothing on the surface of the earth. I illustrated this at lower resolution, and suggested extensions that would restrict smoothing to separate regions such as land and water, and deal with the fractional edges between.

The hexagon grids were very appealing to me, but they were not good enough to make it into main stream use. NASA did not like the way the hexagons overlapped between consecutive resolutions. That was a killer. NASA also needed very fast indexing and professional software to address their massive amounts of data. Any replacement of the the existing gridding scheme would involve a huge effort and have to meet high production standards usually not considered in an academic setting.

My work was reduced to producing icosahedron ornaments using random numbers, smoothing, and color ramps. However I have continued in my efforts to promote the use of round regions. This include applications in higher dimensions. I note that we use little cubes to visualize brains, hearts and livers.   I suggest the use of body-centered cube lattice in 3-D as the basis 3-D binning and 3-D images reconstruction and the use of 4-D body-center hypercube lattice in 4-D for 4-D data (four variables). The near neighbor regions are truncated octahedrons and 24-cells respectively.   The quantizing literature indicates the information loss for using prickly cubes the have from 6 to 26 neighbors depending on if faces, edges or vertices are shared.

Tony Olson continued to involve researcher in addressing his graphics of develop a coherent approach to geospatial sampling and analysis. This methodology has been used by various states and nations.   The methodoly was made available in the R package in **spsurvey (Spatial Survey Design and Analysis)** in March 2007.

The death of the hexagon idea was not the end of the global grid efforts. Kevin Sahr and Denis White observed that an icosahedron has triangle faces that can be recursive partitioned into strictly nested triangles. They studied what happens when using great arc to partition instead partition in the projection. Research lead to different indexing approaches. For example triangle pairs can be joined into quadrilaterals for fast two subscript indexing. A conference emerged to address global grid issues. Hopefully progress has been made building a foundation for future change. Global warming will draw more attention to global modeling.